UNCLASSIFIED

AD ____________

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION ALEXANDRIA, VIRGINIA

DOWNGRADED AT 3 YEAR INTERVALS:
DECLASSIFIED AFTER 12 YEARS
DOD DIR 5200.10

UNCLASSIFIED

6

OFFICE OF NAVAL RESEARCH

Contract N7onr-35810

NR-060-063

Technical Report No. 10

THE REINFORCEMENT TO FULL STRENGTH OF A THIN SLAB WITH A SLIT

by

P. G. Hodge, Jr. and R. K. Froyd

GRADUATE DIVISION OF APPLIED MATHEMATICS

BROWN UNIVERSITY

PROVIDENCE, R. I.

January, 1953

The Reinforcement to Full Strength of a Thin Slab with a Slit[1]

By P. G. Hodge, Jr.[2] and R. K. Froyd[3]

Abstract. A thin slab with a centered slit parallel to an edge of the slab is subjected to uniform tensile stresses across its edges. The material in the slab is assumed to be perfectly plastic and to satisfy Tresca's yield condition. A square reinforcement is designed so that the slit slab will not yield under any load which could be supported by the same slab without a slit.

1. Introduction. Before considering the problem stated in the abstract, it will prove convenient to determine the yield load of a square slab of side 2 and thickness h with a centered slit of length 2a (Fig. 1). In Sec. 6, it will then be shown that once this problem is solved, the design of a reinforcement for full strength can be completed by elementary calculations.

The analysis will utilize two theorems of Drucker, Greenberg, and Prager [1].[4] Consider first the following definitions.

1. The results presented in this paper were obtained in the course of research conducted under Contract N7onr-35810 between the Office of Naval Research and Brown University.

2. Assistant Professor of Mathematics, University of California, Los Angeles, Calif.

3. Graduate Student, University of California, Los Angeles, Calif.

4. Numbers in square brackets refer to references collected at the end of the paper.

a. A statically admissible stress field is a stress field which is piecewise continuous and has continuous tractions across any surface of discontinuity, is in equilibrium with applied loads, and nowhere involves a shearing stress greater than the yield stress k.

b. The internal rate of dissipation of energy D_i is the product of the absolutely greatest principal plastic strain rate by the yield stress:

$$D_i = 2k \max |\dot{\varepsilon}|.$$

c. The external rate of dissipation of energy D_e is the rate at which the applied loads do work on their points of application:

$$D_e = \underline{F} \cdot \underline{v}\,.$$

d. A kinematically admissible velocity field is any incompressible velocity field for which the total internal rate of dissipation of energy does not exceed the external rate:

$$\int_V D_i \, dV \leq \int_S D_e \, dS.$$

With these definitions, the theorems of Drucker, Greenberg, and Prager state that:

1. The collapse load is the largest load for which it is possible to find a statically admissible stress field;

2. The collapse load is the smallest load for which it is possible to find a kinematically admissible velocity field.

Observe that any statically admissible stress field furnishes a lower bound on the collapse load, while any kinematically admissible velocity field furnishes an upper bound. In particular, if both a statically admissible stress field and a kinematically admissible velocity field can be found for the same load, this load is the collapse load.

2. Uniaxial tension, upper bound. An upper bound for the slit slab loaded in uniaxial tension perpendicular to the slit can be found by the method used by Hodge and Prager [2] in the discussion of a slab with a circular cutout. To make the present paper self contained the analysis is repeated here.

Let the applied stress be $2vk$ and consider a discontinuous velocity field in which half of the slab remains at rest while the other half moves out of the plane at 45°, as a rigid body with a velocity v. Figure 2 shows a cross-section of the slab for such a motion. This may be considered as the limiting case of the continuous velocity field shown in Fig. 3.[5] In the transition region of width β the absolutely greatest principal strain rate is $v/2\beta$; in the remainder of the slab the strain rates vanish.

The total internal rate of dissipation of energy $\mathcal{D}_1$ is given by

5. The triangular regions having an area of $h^2/2$ are neglected.

$$\mathcal{D}_i = \int_V D_i dV = 2\sqrt{2}\, kvh(1-a). \tag{2.1}$$

Since this is independent of β it remains valid in the limit as $\beta \longrightarrow 0$.

The total external rate of dissipation of energy $\mathcal{D}_e$ is given by

$$\mathcal{D}_e = \int_S \underline{F} \cdot \underline{v}\, dS = 2\sqrt{2}\, vkhv. \tag{2.2}$$

For this velocity field to be kinematically admissible the total external rate of dissipation of energy must be greater than or equal to the internal rate. Thus $\mathcal{D}_e \geq \mathcal{D}_i$, or, in view of Eqs. (2.1) and (2.2)

$$v \geq 1 - a. \tag{2.3}$$

Therefore, the collapse load of the slab under uniaxial tension is less than or equal to $2k(1-a)h$ per unit length.

3. Uniaxial tension, lower bound. A state of plane stress can be described by specifying the two non-vanishing principal stress components and a principal direction. Let θ be the smallest non-negative angle between the x axis and a principal direction, s the principal stress across an element inclined at an angle θ to the x axis, and r the other principal stress. The reduced mean normal stress ω and the reduced difference between the principal stresses χ are then defined by

$$\omega = \frac{1}{2k}(r+s), \qquad \chi = \frac{1}{2k}(r-s). \tag{3.1}$$

The normal and tangential tractions across an element inclined

at an angle α to the x axis are given in terms of ω, χ, and θ by

$$(N/k) = \omega - \chi \cos 2(\theta-\alpha), \quad (T/k) = - \chi \sin 2(\theta-\alpha). \quad (3.2)$$

Consider a discontinuous stress field of the type shown in Fig. 4. Due to symmetry, it is necessary to consider only one quarter of the slab. Across AB, the normal traction must equal the applied load $2\mu kh$ and the shearing traction must vanish. In terms of the variables θ, r, and s this requires that $\theta_1 = 0$ and $s_1 = 2\mu k$. The stress r_1 is undetermined. Similarly the fact that no tractions are transmitted across the slit OD, or the side BC leads to $\theta_4 = s_4 = \theta_3 = r_3 = 0$. Finally, overall equilibrium of the slab requires that $s_3 = 2\mu k/(1-a)$.

The interior components of stress r_1 and r_4 and the three components in region 2 must now be determined. Let $y = \frac{r_1}{2k}$, $z = \frac{r_4}{2k}$, $\omega = \omega_2$, $\chi = \chi_2$, $\theta = \theta_2$, and let α_{ij} be the angle between the x axis and the line of discontinuity separating regions i and j. From the geometry of Fig. 4,

$$\begin{aligned} \tan \alpha_{12} &= 1 - \delta, \\ \tan \alpha_{23} &= \frac{1}{1-a}, \qquad (3.3) \\ \tan \alpha_{24} &= - \frac{\delta}{a}. \end{aligned}$$

The continuity of normal and tangential tractions across the lines of discontinuity requires that the following six equations be satisfied:

$$\begin{aligned}
\omega - \chi \cos 2(\theta-\alpha_{12}) &= y + \mu - (y-\mu)\cos 2\alpha_{12}, \\
\omega - \chi \cos 2(\theta-\alpha_{23}) &= \frac{\mu}{1-a}(1 + \cos 2\alpha_{23}), \\
\omega - \chi \cos 2(\theta-\alpha_{24}) &= z(1 - \cos 2\alpha_{24}), \\
\chi \sin 2(\theta-\alpha_{12}) &= -(y-\mu)\sin 2\alpha_{12}, \qquad (3.4)\\
\chi \sin 2(\theta-\alpha_{23}) &= \frac{\mu}{1-a} \sin 2\alpha_{23}, \\
\chi \sin 2(\theta-\alpha_{24}) &= - z \sin 2\alpha_{24}.
\end{aligned}$$

Since s_3 was determined so as to satisfy an overall equilibrium condition, only five of Eqs. (3.4) are independent. Their solutions can be seen to be

$$\begin{aligned}
\tan 2\theta &= \frac{2a}{\delta + a(1-a)}, \\
\chi &= -\frac{\mu\sqrt{[\delta + a(1-a)]^2 + 4a^2}}{a + \delta(1-a)} \\
\omega &= \frac{\mu[\delta - a(1-a)]}{a + \delta(1-a)} \qquad (3.5)\\
y &= \frac{\mu a}{1 - \delta} \\
z &= -\frac{\mu a}{\delta}.
\end{aligned}$$

As a check it will be noted that the total traction across OA vanishes.

In the present notation the yield condition of Tresca may be written

$$|r| \leq 2k, \quad |s| \leq 2k, \quad |\chi| \leq 1. \qquad (3.6)$$

If the above stress field is to be statically admissible the six Inequalities (3.6) must be satisfied throughout the plate. In each of the four regions the <u>governing inequality</u> is defined as that one which would be violated by the smallest value of μ. The substitution of Eqs. (3.5) into the governing inequality of each region leads to

$$\frac{\mu a}{1 - \delta} \leq 1, \tag{3.7}$$

$$\frac{\mu\sqrt{[\delta + a(1-a)]^2 + 4a^2}}{a + \delta(1-a)} \leq 1, \tag{3.8}$$

$$\frac{\mu}{1 - a} \leq 1, \tag{3.9}$$

$$\frac{\mu a}{\delta} \leq 1. \tag{3.10}$$

Solving Inequalities (3.7), (3.8), and (3.10) for δ one obtains

$$\delta \leq 1 - \mu a, \tag{3.11}$$

$$\frac{-a(1-a)(\mu^2-1)-\sqrt{a^2(1-a)^2(\mu^2-1)^2-[\mu^2-(1-a)^2]\{\mu^2[a^2(1-a)^2+4a^2]-a^2\}}}{\mu^2 - (1-a)^2} \leq \delta, \tag{3.12}$$

$$\mu a \leq \delta . \tag{3.13}$$

In addition it follows from the geometry of Fig. 4 that

$$0 \leq \delta \leq 1. \tag{3.14}$$

Except for the restrictions of Inequalities (3.11) through (3.14), δ is arbitrary. A necessary and sufficient

condition that there exists some value of δ satisfying all these inequalities is that each lower bound on δ be less than or equal to each upper bound. Since μ and a are positive, the resulting six inequalities are all satisfied provided that

$$\mu \leq \frac{1}{2a}, \tag{3.15}$$

$$f(\mu) = a^2\mu^4 - 2a(1+a-a^2)\mu^3 + (1+2a+2a^2)\mu^2 + 2a(1-a)\mu - 1 \leq 0. \tag{3.16}$$

Further, Inequality (3.15) is seen to be satisfied whenever (3.9) is satisfied.

The polynomial $f(\mu)$ is increasing in the interval $0 \leq \mu \leq 1$. Indeed, since

$$f'(\mu) = 4a^2\mu^3 - 6a(1+a-a^2)\mu^2 + 2(1+2a+2a^2)\mu + 2a(1-a), \tag{3.17}$$

and since for $0 \leq a \leq 1$, it is easily shown that

$$2(1+2a+2a^2) > 6a(1+a-a^2), \tag{3.18}$$

the derivative is positive and the conclusion follows. Thus, in the interval $0 \leq \mu \leq 1 - a$,

$$f(\mu) \leq f(1-a) = -a^3(2+2a-4a^2+a^3) < 0, \tag{3.19}$$

so that (3.16) is satisfied by any value of μ satisfying (3.9).

Therefore, the above stress field will be statically admissible provided only that the load is less than or equal to $2k(1-a)h$ per unit length. Since this is the same as the upper bound found in the previous section, the collapse load per unit length under uniaxial loading perpendicular to the slit is

$$hTy = 2k(1-a)h. \tag{3.20}$$

4. Uniform biaxial tension. The upper bound obtained in Sec. 2 is also valid for the case of biaxial tension, since the side loads do no work. A lower bound is obtained by the same method as in Sec. 3. The stress field is shown in Fig. 5, where the values of the quantities which may be determined by inspection are specified. With the definitions of Sec. 3, the equilibrium conditions across the lines of discontinuity lead to

$$\begin{aligned}
\omega - \chi \cos 2(\theta-\alpha_{12}) &= y + \mu - (y-\mu)\cos 2\alpha_{12}, \\
\omega - \chi \cos 2(\theta-\alpha_{23}) &= \frac{\mu(2-a)}{1-a} + \frac{\mu a}{1-a} \cos 2\alpha_{23}, \\
\omega - \chi \cos 2(\theta-\alpha_{24}) &= z(1 - \cos 2\alpha_{24}), \\
\chi \sin 2(\theta-\alpha_{12}) &= - (y-\mu)\sin 2\alpha_{12}, \\
\chi \sin 2(\theta-\alpha_{23}) &= \frac{\mu a}{1-a} \sin 2\alpha_{23}, \\
\chi \sin 2(\theta-\alpha_{24}) &= - z \sin 2\alpha_{24}.
\end{aligned} \tag{4.1}$$

The solution of Eqs. (4.1) is found to be

$$\begin{aligned}
\tan 2\theta &= \frac{2}{\delta-a}, \\
\chi &= - \mu \frac{a\sqrt{(\delta-a)^2 + 4}}{a + \delta(1-a)}, \\
\omega &= \mu \frac{a^2 + \delta(2-a)}{a + \delta(1-a)}, \\
y &= \mu \frac{a + 1 - \delta}{1 - \delta}, \\
z &= \mu \frac{\delta - a}{\delta}.
\end{aligned} \tag{4.2}$$

In regions 1, 3, and 4 the governing inequalities are found to be

$$\mu \frac{a + 1 - \delta}{1 - \delta} \leq 1,$$

$$\mu \frac{1}{1 - a} \leq 1, \tag{4.3}$$

$$\mu \frac{|\delta - a|}{\delta} \leq 1,$$

while in region 2,

$$\mu \frac{a\sqrt{(\delta-a)^2 + 4}}{a + \delta(1-a)} \leq 1 \quad \text{if} \quad 0 \leq \delta \leq a, \tag{4.4}$$

but

$$\frac{\mu}{2} \cdot \frac{a^2 + \delta(2-a) + a\sqrt{(\delta-a)^2 + 4}}{a + \delta(1-a)} \leq 1 \quad \text{if} \quad a \leq \delta \leq 1. \tag{4.5}$$

It is found that if δ is taken equal to a, the resulting inequalities are all satisfied provided that

$$\mu \leq 1 - a. \tag{4.6}$$

Since this is the same as for the upper bound, the collapse load per unit length for uniform biaxial loading is

$$hT_x = hT_y = 2k(1-a)h. \tag{4.7}$$

It is interesting to note that the stress field obtained by setting $\delta = a$ in Eqs. (4.2) yields

$$r_4 = 2kz = 0 \tag{4.8}$$

so that region 4 is stress free. It follows that the yield load for a square slab with a square diamond cut-out (Fig. 6) under biaxial tension is also given by Eq. (4.7).

5. Other loadings. If the slab is loaded in uniaxial tension parallel to the slit, the slit will have no effect, so that the collapse load is

$$hT_x = 2kh > 2k(1-a)h \qquad (5.1)$$

per unit length. Further, statically admissible stress fields obviously exist for compressive loads which are equal in magnitude to the admissible tensile loads already considered.

An argument used by Hodge [3] now shows that if (hS_x, hS_y) represents any load per unit length which can be carried without yielding by the unslit slab, the slab with a slit will support the load per unit length

$$hT_x = S_x(1-a)h, \qquad hT_y = S_y(1-a)h. \qquad (5.2)$$

To see this, observe first that the loads S_x, S_y may be represented in a loading space by the hexagon ABCDEF (Fig. 7). Thus, it is desired to show that for any load in the hexagon abcdef a statically admissible stress field exists.

Any point on the hexagon abcdef can be represented as a linear combination of some two of the vertices of the form

$$\underline{X} = \beta[\alpha\underline{a}_1 + (1-\alpha)\underline{a}_2] \quad 0 \leq \alpha \leq 1, \quad 0 \leq \beta \leq 1. \qquad (5.3)$$

Here $\underline{X}$ represents the vector from the origin to a point on the hexagon and $\underline{a}_1$ and $\underline{a}_2$ represent vectors from the origin to some two of the six vertices. Since statically admissible stress distributions exist for the loads represented by the vertices, if at any point in the slab, for the load represented by $\underline{X}$, the stress

distribution corresponding to (5.3) is taken, it follows from the convexity of the yield surface that the resulting stress distribution will be statically admissible.

6. Reinforcement of the slab. In this section it will be shown how the results previously obtained can be used to determine the dimensions of square reinforcements welded to the sides of the slab so that the reinforced slab (Fig. 8) will be able to carry any load which could be carried by the unslit slab. The reinforced slab is assumed to be in a state of generalized plane stress.

Let the edges of the slab be loaded with loads hS_x, hS_y per unit length (Fig. 8). In the part of the slab not covered by the reinforcement, the constant stress field

$$\sigma_x = S_x, \qquad \sigma_y = S_y \tag{6.1}$$

is in equilibrium with the given loads, and will be statically admissible if the unslit slab could carry the given loads. This stress field will transmit the tractions hS_x, hS_y per unit length to the reinforced part of the slab.

From the previous section, it is known that the reinforced part will not yield provided that the edge loads per unit length are not greater than $(\frac{b-a}{b})HS_x$, $(\frac{b-a}{b})HS_y$. Therefore, it follows that a statically admissible plane stress field for the entire slab can be found, provided that

$$\frac{H}{h} \geq \frac{b}{b-a} . \tag{6.2}$$

To show that this is the least reinforcement which will be safe for all loadings, it is necessary to consider only the kinematically admissible velocity field associated with uniaxial loading perpendicular to the slit. An analysis similar to that of Sec. 2 shows that for a load hTy,

$$\frac{H}{h} \leq \frac{b}{b-a} + \frac{(Ty/2k) - 1}{b-a} \, . \tag{6.3}$$

In particular, if Ty = 2k (full strength), then Inequalities (6.2) and (6.3) show that the thickness ratio for a square of given side b is

$$\frac{H}{h} = \frac{b}{b-a} \, . \tag{6.4}$$

References

1. D. C. Drucker, W. Prager, and H. J. Greenberg, Extended limit design theorems for continuous media, Q. Appl. Math. 9, (1952).

2. P. G. Hodge, Jr., and W. Prager, Limit design of reinforcements of cut-outs in slabs, Tech. Rep. B11-2, Contract N7onr-35810, Brown University, Providence, R. I., (1951).

3. P . G. Hodge, Jr., Upper and lower bounds on the yield load of a square slab with a centered circular cut-out, Tech. Rep. B11-7, Contract N7onr-35810, Brown University, Providence, R. I., (1952).

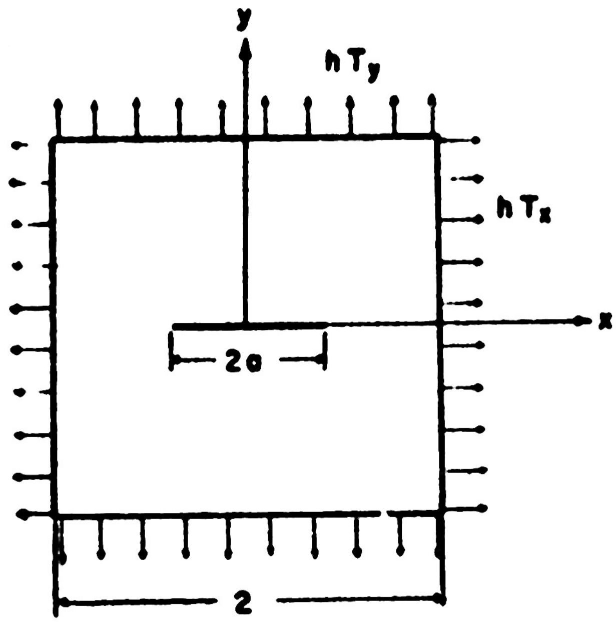

Fig. 1. Unreinforced slab with slit.

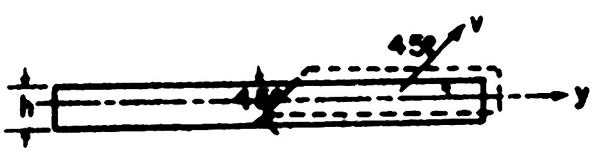

Fig. 2. Discontinuous velocity field.

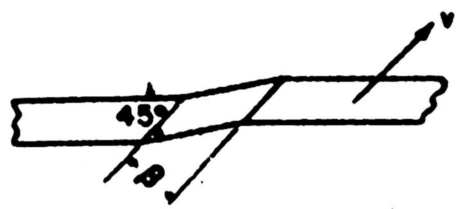

Fig. 3. Continuous velocity field.

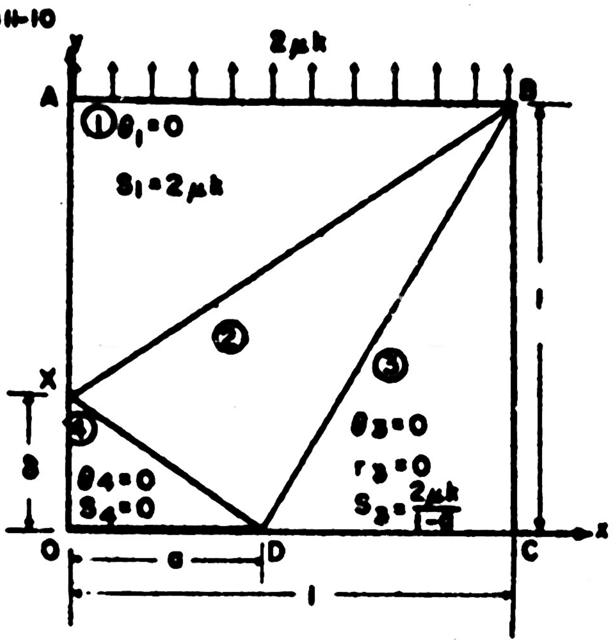

Fig. 4. Discontinuous stress field, uniaxial tension.

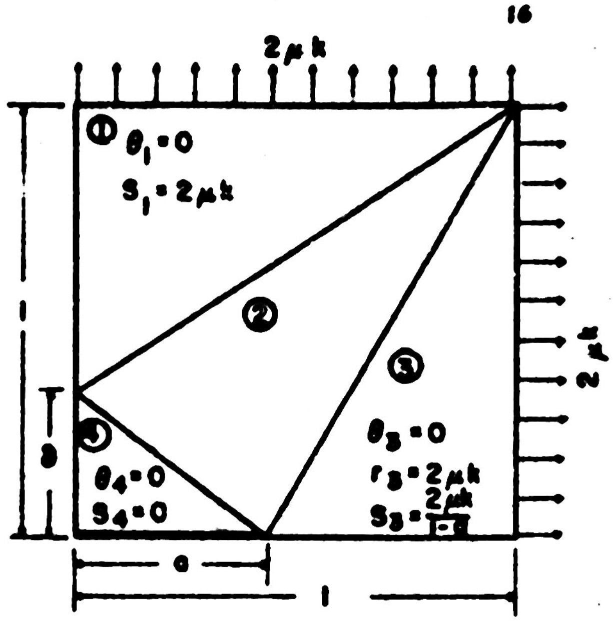

Fig. 5. Discontinuous stress field, biaxial tension.

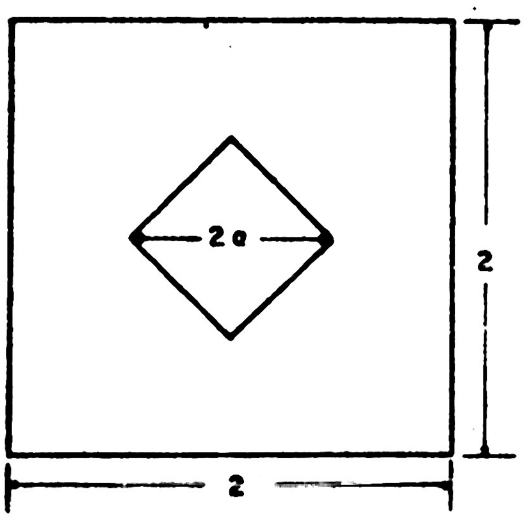

Fig. 6. Slab with diamond cut-out.

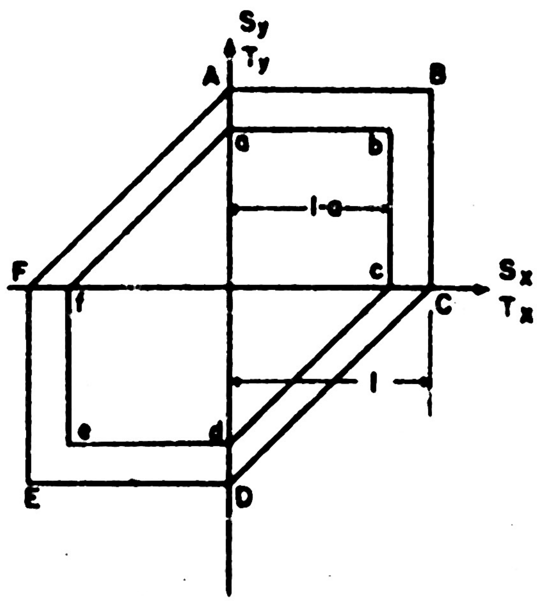

Fig. 7. Load hexagons.

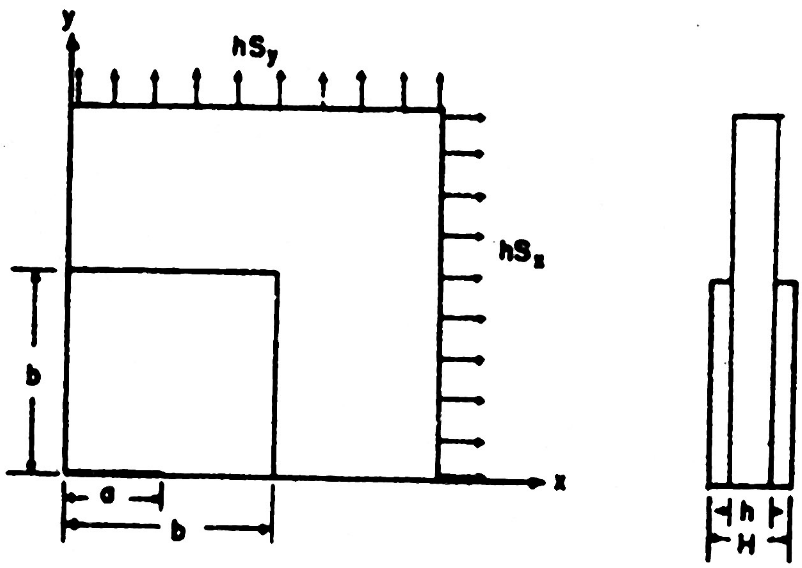

Fig. 8. Reinforced slab.

Distribution List
for
Technical and Final Reports Issued Under
Office of Naval Research Project NR-360-364, Contract N7onr-35810

I: Administrative, Reference and Liaison Activities of ONR

Chief of Naval Research
Department of the Navy
Washington 25, D. C.
Attn: Code 438 (2)
Code 432 (1)
Code 466(via Code 108)(1)

Director, Naval Research Lab.
Washington 25, D. C.
Attn: Tech. Info. Officer (9)
Technical Library (1)
Mechanics Division (2)

Commanding Officer
Office of Naval Research
Branch Office
495 Summer Street
Boston 10, Massachusetts (2)

Commanding Officer
Office of Naval Research
Branch Office
346 Broadway
New York 13, New York (1)

Commanding Officer
Office of Naval Research
Branch Office
1000 Geary Street
San Francisco, California (1)

Commanding Officer
Office of Naval Research
Branch Office
1030 Green Street
Pasadena, California (1)

Officer in Charge
Office of Naval Research
Branch Office, London
Navy No. 100
FPO, New York, N. Y. (5)

Library of Congress
Washington 25, D. C.
Attn: Navy Research Section (2)

Commanding Officer
Office of Naval Research
Branch Office
844 N. Rush Street
Chicago 11, Illinois (1)

II: Department of Defense and other interested Government Activities

a) General

Research & Development Board
Department of Defense
Pentagon Building
Washington 25, D. C.
Attn: Library(Code 3D-1075) (1)

Armed Forces Special Weapons Project
P. O. Box 2610
Washington, D. C.
Attn: Lt. Col. G.F.Blunda (2)

Joint Task Force 3
12 St. & Const. Ave., N.W.
(Temp. U)
Washington 25, D. C.
Attn: Major B.D.Jones (1)

b) Army

Chief of Staff
Department of the Army
Research & Development Division
Washington 25, D. C.
Attn: Chief of Res. & Dev. (1)

Office of the Chief of Engineers
Assistant Chief for Works
Department of the Army
Bldg. T-7, Gravelly Point
Washington 25, D. C.
Attn: Structural Branch
(R. L. Bloor) (1)

Engineering Research & Development Laboratory
Fort Belvoir, Virginia
Attn: Structures Branch (1)

Army (cont.)

Office of the Chief of Engineers
Asst. Chief for Military
Construction
Department of the Army
Bldg. T-3, Gravelly Point
Washington 25, D. C.
Attn: Structures Branch
(M. F. Carey) (1)
Protective Construction
Branch (I.O.Thornley) (1)

Office of the Chief of Engineers
Asst. Chief for Military
Operations
Department of the Army
Bldg T-7, Gravelly Point
Washington 25, D. C.
Attn: Structures Development
Branch (W.F.Woollard) (1)

U. S. Army Waterways Experiment
Station
P. O. Box 631
Halls Ferry Road
Vicksburg, Mississippi
Attn: Col. H. J. Skidmore (1)

The Commanding General
Sandia Base, P.O.Box 5100
Albuquerque, New Mexico
Attn: Col. Canterbury (1)

Operations Research Officer
Department of the Army
Ft. Lesley J. McNair
Washington 25, D. C.
Attn: Howard Brackney (1)

Office of Chief of Ordnance
Office of Ordnance Research
Department of the Army
The Pentagon Annex #2
Washington 25, D. C.
Attn: ORDTB-PS (1)

Ballistics Research Laboratory
Aberdeen Proving Ground
Aberdeen, Maryland
Attn: Dr. C. W. Lampson (1)

c) Navy

Chief of Naval Operations
Department of the Navy
Washington 25, D.C.
Attn: OP-31 (1)
OP-363 (1)

Chief of Bureau of Ships
Department of the Navy
Washington 25, D. C.
Attn: Director of Research (2)
Code 423 (1)
Code 442 (1)
Code 421 (1)

Director, David Taylor Model Basin
Department of the Navy
Washington 7, D. C.
Attn: Code 720, Structures
Division (1)
Code 740, Hi-Speed
Dynamics Div. (1)

Commanding Officer
Underwater Explosions Research Div.
Code 290
Norfolk Naval Shipyard
Portsmouth, Virginia (1)

Commander
Portsmouth Naval Shipyard
Portsmouth, N. H.
Attn: Design Division (1)

Director, Materials Laboratory
New York Naval Shipyard
Brooklyn 1, New York (1)

Chief of Bureau of Ordnance
Department of the Navy
Washington 25, D. C.
Attn: Ad-3, Technical Library (1)
Rec, P. H. Girouard (1)

Naval Ordnance Laboratory
White Oak, Maryland
RFD 1, Silver Spring, Maryland
Attn: Mechanics Division (1)
Explosive Division (1)
Mech. Evaluation Div. (1)

Commander
U. S. Naval Ordnance Test Station
Inyokern, California
Post Office - China Lake, Calif.
Attn: Scientific Officer (1)

Naval Ordnance Test Station
Underwater Ordnance Division
Pasadena, California
Attn: Structures Division (1)

Navy (cont.)

Chief of Bureau of Aeronautics
Department of the Navy
Washington 25, D. C.
Attn: TD-41, Technical Library (1)

Chief of Bureau of Ships
Department of the Navy
Washington 25, D. C.
Attn: Code P-314 (1)
Code C-313 (1)

Officer in Charge
Naval Civil Engr. Research & Evaluation Laboratory
Naval Station
Port Hueneme, California (1)

Superintendent
U.S. Naval Post Graduate School
Annapolis, Maryland (1)

d) Air Forces

Commanding General
U. S. Air Forces
The Pentagon
Washington 25, D. C.
Attn: Res. & Development Div. (1)

Deputy Chief of Staff, Operations
Air Targets Division
Headquarters, U. S. Air Forces
Washington 25, D. C.
Attn: AFOIN-T/PV (1)

Office of Air Research
Wright-Patterson Air Force Base
Dayton, Ohio
Attn: Chief, Applied Mechanics Group (1)

e) Other Government Agencies

U. S. Atomic Energy Commission
Division of Research
Washington, D. C. (1)

Director, National Bureau of Standards
Washington, D. C.
Attn: Dr. W. H. Ramberg (1)

Supplementary Distribution List

Addressee	No. of Copies: Unclassified Reports	No. of Copies: Classified Reports
Professor Lynn Beedle, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania	1	-
Professor R. L. Bisplinghoff, Dept. of Aeronautical Engineering, Massachusetts Institute of Technology, Cambridge 39, Massachusetts	1	1
Professor Hans Bleich, Dept. of Civil Engineering, Columbia University, Broadway at 117th St., New York 27, N. Y.	1	1

Addressee	Unclassifed Reports	Classified Reports
Professor B. A. Boley Dept. of Aeronautical Engineering Ohio State University Columbus, Ohio	1	-
Professor G. F. Carrier Graduate Division of Applied Mathematics Brown University Providence, R. I.	1	1
Professor R. J. Dolan Dept. of Theoretical & Applied Mechanics University of Illinois Urbana, Illinois	1	-
Professor Lloyd Donnell Department of Mechanics Illinois Institute of Technology Technology Center Chicago 16, Illinois	1	-
Professor A. C. Eringen Illinois Institute of Technology Department of Mechanics Technology Center Chicago 16, Illinois	1	-
Professor B. Fried Dept. of Mechanical Engineering Washington State College Pullman, Washington	1	-
Mr. Martin Goland Midwest Research Institute 4049 Pennsylvania Avenue Kansas City 2, Missouri	1	-
Dr. J. N. Goodier School of Engineering Stanford University Stanford, California	1	-
Professor R. M. Hermes College of Engineering University of Santa Clara Santa Clara, California	1	1
Professor R. J. Hansen Dept. of Civil & Sanitary Engineering Massachusetts Institute of Technology Cambridge 39, Massachusetts	1	1

Addressee	Unclassified Reports	Classified Reports
Professor M. Hetenyi Walter P. Murphy Professor Northwestern University Evanston, Illinois	1	-
Dr. N. J. Hoff, Head Department of Aeronautical Engineering & Applied Mechanics Polytechnic Institute of Brooklyn 99 Livingston Street Brooklyn 2, New York	1	1
Dr. J. H. Hollomon General Electric Research Laboratories 1 River Road Schenectady, New York	1	-
Dr. W. H. Hoppmann Department of Applied Mechanics Johns Hopkins University Baltimore, Maryland	1	1
Professor L. S. Jacobsen Department of Mechanical Engineering Stanford University Stanford, California	1	1
Professor J. Kempner Department of Aeronautical Engineering and Applied Mechanics Polytechnic Institute of Brooklyn 99 Livingston Street Brooklyn 2, New York	1	1
Professor George Lee Department of Aeronautical Engineering Renssalaer Polytechnic Institute Troy, New York	1	-
Professor Paul Lieber Department of Aeronautical Engineering Renssalaer Polytechnic Institute Troy, New York	1	1
Professor Glen Murphy, Head Department of Theoretical & Applied Mechanics Iowa State College Ames, Iowa	1	-
Professor N. M. Newmark Department of Civil Engineering University of Illinois Urbana, Illinois	1	1

Distribution List

Addressee	Unclassified Reports	Classified Reports
Professor Jesse Ormondroyd University of Michigan Ann Arbor, Michigan	1	-
Dr. W. Osgood Armour Research Institute Technology Center Chicago, Illinois	1	-
Dr. R. P. Petersen, Director Applied Physics Division Sandia Laboratory Albuquerque, New Mexico	1	1
Dr. A. Phillips School of Engineering Stanford University Stanford, California	1	-
Dr. W. Prager, Chairman Graduate Division of Applied Mathematics Brown University Providence 12, R. I.	1	1
Dr. S. Raynor Armour Research Foundation Illinois Institute of Technology Chicago, Illinois	1	-
Professor E. Reissner Department of Mathematics Massachusetts Institute of Technology Cambridge 39, Massachusetts	1	-
Professor M. A. Sadowsky Illinois Institute of Technology Technology Center Chicago 16, Illinois	1	-
Professor V. L Salerno Department of Aeronautical Engineering Renssalaer Polytechnic Institute Troy, New York	1	1
Professor M. G. Salvadori Department of Civil Engineering Columbia University Broadway at 117th Street New York 27, New York	1	-
Professor J. E Stallmeyer Talbot Laboratory Department of Civil Engineering University of Illinois Urbana, Illinois	1	1

Distribution List

Addressee	Unclassified Reports	Classified Reports
Professor E. Sternberg Illinois Institute of Technology Technology Center Chicago 16, Illinois	1	-
Professor R. G. Sturm Purdue University Lafayette, Indiana	1	-
Professor F. K. Teichmann Department of Aeronautical Engineering New York University University Heights, Bronx New York, N. Y.	1	-
Professor C. T. Wang Department of Aeronautical Engineering New York University University Heights, Bronx New York, N. Y.	1	-
Project File	2	2
Project Staff	5	-
For possible future distribution by the University	10	-
To ONR Code 438, for possible future distribution	-	10